PERIOD-V

M10CH9

TOPIC 9

Vector in a Plane

9.1 SCALAR *AND* VECTOR QUANTITIES

Quantities are of two types, namely scalars and vectors.

Scalars

A quantity that has only magnitude is called a *scalar.*

For example: Mass, length, time, temperature, area, volume, speed, density etc. are scalars.

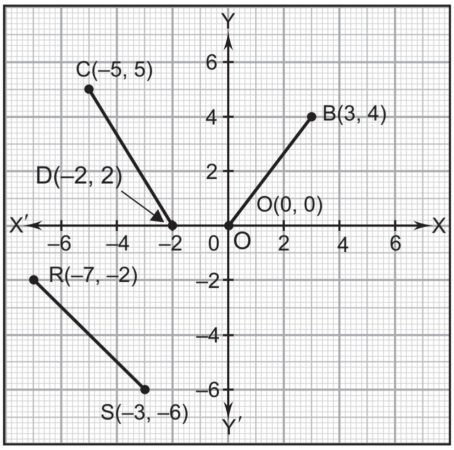
Vectors

A quantity that has magnitude as well as a direction is called a *vector.*

For example: Displacement, velocity, acceleration, force etc. are vectors.

Representation of Vectors

A *vector* is any physical quantity that has *length* (or magnitude) and *direction.* In other words, vectors represent the length and directions of lines in the number plane.



For example: $\overrightarrow{AB} = \begin{pmatrix}3\\4\end{pmatrix}$,

$\overrightarrow{CD} = \begin{pmatrix}-5\\6\end{pmatrix}$, $\mathbf{a} = \begin{pmatrix}2\\-1\end{pmatrix}$ etc.

Notice that vectors are expressed in columns, one value on top of the other. The values at the top are called the *x-components* of the vectors, while the values at the bottom are called the *y-components.*

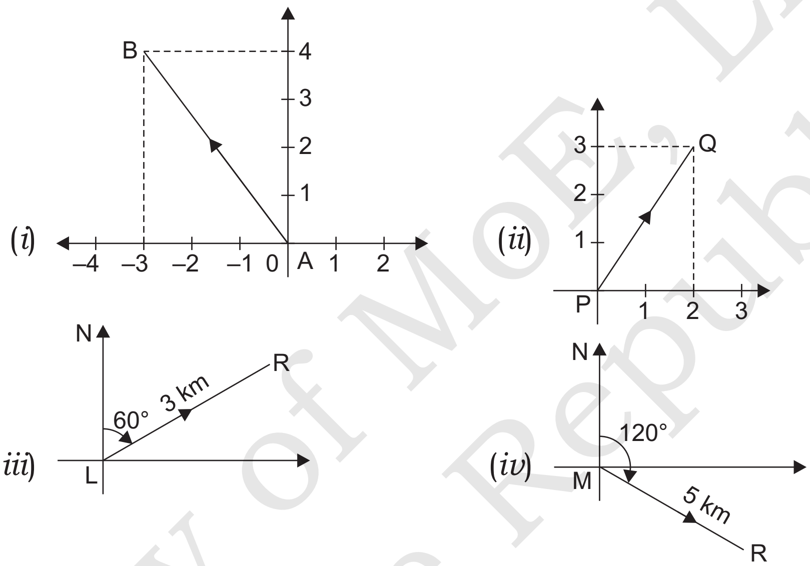
(See vectors $\overrightarrow{RS}$ and $\overrightarrow{CD}$ and $\overrightarrow{OB}$ in the figure). Note that the arrow placed on the vectors show the direction of the vectors.

Example 1. *Express the following vectors graphically:*

(*i*) $\overrightarrow{AB} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$ (*ii*) $\overrightarrow{PQ} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

(*iii*) $\overrightarrow{LR}$ = (3 km, 060°) (*iv*) $\overrightarrow{MR}$ = (5 km, 120°)

Solution.



EXERCISE 9.1

1. Express the following vectors graphically.

   (*a*) $\overrightarrow{PQ}$ = (4 km, 30°) (*b*) $\overrightarrow{LM} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

2. Mark the following vectors on a graph sheet.

   (*a*) $\overrightarrow{OP} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$; (*b*) $\overrightarrow{OQ} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ (*c*) $\overrightarrow{OR} = \begin{pmatrix} -2 \\ -3 \end{pmatrix}$; (*d*) $\overrightarrow{OS} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$

3. (*a*) Represent graphically a displacement of 40 km, 30° east of north.

   (*b*) Represent graphically a displacement of 40 km, 30° west of south.

4. Classify the following as scalar and vectors:

(*a*) speed (*b*) velocity (*c*) mass (*d*) temperature
(*e*) acceleration (*f*) weight (*g*) time period (*h*) distance
(*i*) force (*j*) workdone.

9.2 TYPES OF VECTOR QUANTITIES

Zero Vector

A *zero vector* is a vector which *begins* and *ends* at the *same point.* Their length as well as direction is always zero.

For example: If P(2, 3) and Q(2, 3), then vector

$$\overrightarrow{PQ} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \text{ i.e. } \overrightarrow{PQ} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Position Vector

A *position vector* is a vector which begins from the *origin* and ends at a point. $\overrightarrow{OB}$ is a vector see in the figure previous section 9.1, (*Page no. 170*).

For example: If P(4, 5), then $\overrightarrow{OP} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$ is the position vector. That is,

$$\overrightarrow{OP} = \begin{pmatrix} 4 \\ 5 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$$

Example 2. *Find $\overrightarrow{PQ}$ and hence $\overrightarrow{QP}$ if P(4, 8) and Q(–1, 5).*

Solution. The position vectors of P and Q referred to the origin O are $\overrightarrow{OP} = \mathbf{p} = \begin{pmatrix} 4 \\ 8 \end{pmatrix}$ and $\overrightarrow{OQ} = \mathbf{q} = \begin{pmatrix} -1 \\ 5 \end{pmatrix}$ respectively.

$$\therefore \quad \overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = \mathbf{p} - \mathbf{q}$$

$$= \begin{pmatrix} -1 \\ 5 \end{pmatrix} - \begin{pmatrix} 4 \\ 8 \end{pmatrix} = \begin{pmatrix} -5 \\ -3 \end{pmatrix}$$

$$\overrightarrow{QP} = -\overrightarrow{PQ} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}.$$

Negative Vector

Negative vector is the *opposite* or *inverse* of a vector.

For example: If $\overrightarrow{PQ} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$ is a vector then vector $\overrightarrow{QP} = -\overrightarrow{PQ} = \begin{pmatrix} -2 \\ -5 \end{pmatrix}$ is the negative or the opposite.

Example 3. (*i*) *If* $\overrightarrow{PQ} = \begin{pmatrix} -3 \\ -4 \end{pmatrix}$, *then find* $\overrightarrow{QP}$.

(*ii*) *If* $\overrightarrow{AB} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$, *then find* $\overrightarrow{BA}$.

Solution. (*i*) We have $\overrightarrow{PQ} = \begin{pmatrix} -3 \\ -4 \end{pmatrix}$

$\therefore$ Negative or opposite of $\overrightarrow{PQ}$ is $\overrightarrow{QP}$

$$\therefore \quad \overrightarrow{QP} = -\overrightarrow{PQ} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

(*ii*) We have $\overrightarrow{AB} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$

$\therefore$ Negative or inverse (opposite) of $\overrightarrow{AB}$ is $\overrightarrow{BA}$

$$\therefore \quad \overrightarrow{BA} = -\overrightarrow{AB} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}.$$

Equal Vectors

If any two or more vectors have the same components, then they are *equal vectors. For example:* $\overrightarrow{AB} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$, $\overrightarrow{BC} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$ are equal vectors.

Example 4. If $\mathbf{p} = \begin{pmatrix} x+3 \\ y-1 \end{pmatrix}$ *and* $\mathbf{q} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$, *find x and y, if* $\mathbf{p} = \mathbf{q}$.

Solution. As $\mathbf{p} = \mathbf{q}$ (given) $\Rightarrow \begin{pmatrix} x+3 \\ y-1 \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$

Equating corresponding components

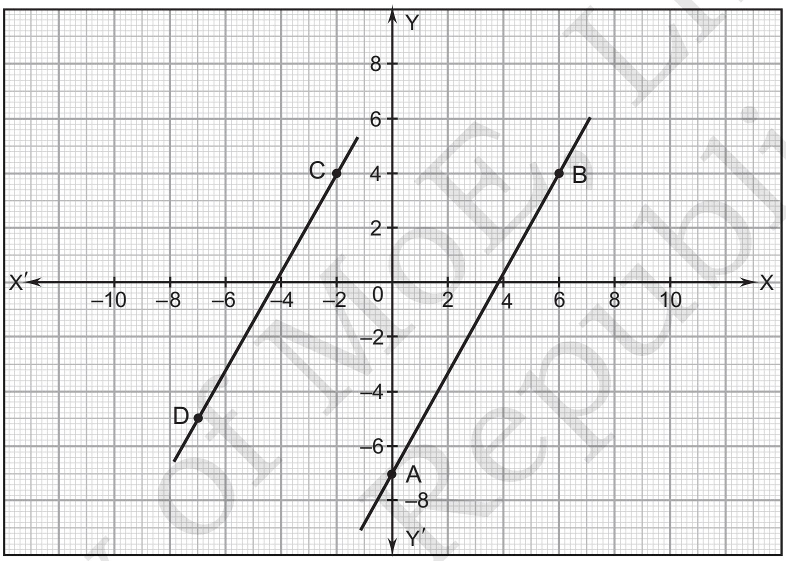
$\Rightarrow \quad x + 3 = -2 \quad \Rightarrow \quad x = -5$

and $\quad y - 1 = 3 \quad \Rightarrow \quad y = 4.$

Parallel Vectors

Two or more vectors are parallel, if one is a *scalar multiple* of the other(s).

For example: In the figure below, vector $\overrightarrow{AB}$ is parallel to vector $\overrightarrow{CD}$. Similarly, vector $\overrightarrow{AB} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$ is parallel to vector $\overrightarrow{CD} = \begin{pmatrix} -3 \\ 9 \end{pmatrix}$ because $3\begin{pmatrix} -1 \\ 3 \end{pmatrix} = \begin{pmatrix} -3 \\ 9 \end{pmatrix}$.



Example 5. *Which of the following is parallel to* $\begin{pmatrix} 20 \\ 8 \end{pmatrix}$?

(*a*) $\begin{pmatrix} -25 \\ 10 \end{pmatrix}$ (*b*) $\begin{pmatrix} -2 \\ -5 \end{pmatrix}$ (*c*) $\begin{pmatrix} -15 \\ -6 \end{pmatrix}$ (*d*) $\begin{pmatrix} -4 \\ -10 \end{pmatrix}$ (*e*) $\begin{pmatrix} 4 \\ -10 \end{pmatrix}$

Solution. The vector which is parallel to $\begin{pmatrix} 20 \\ 8 \end{pmatrix}$ is a scalar multiple of $\begin{pmatrix} 20 \\ 8 \end{pmatrix}$. *i.e.* $k\begin{pmatrix} 20 \\ 8 \end{pmatrix}$ where k is a non-zero number. From the vectors given, $\begin{pmatrix} -15 \\ -6 \end{pmatrix}$ is parallel to $\begin{pmatrix} 20 \\ 8 \end{pmatrix}$ since $k = -\frac{3}{4}$.

Hence, the correct option is (*c*).

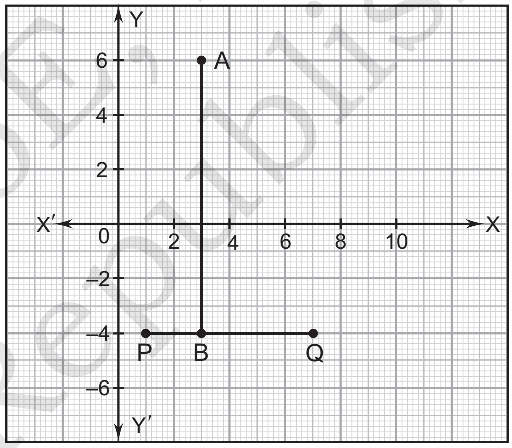
Note: In general, if the vector $\begin{pmatrix} a \\ b \end{pmatrix}$ is parallel to the vector $\begin{pmatrix} c \\ d \end{pmatrix}$, then $a : c = b : d$. That is the ratios of the corresponding components in the same order are equal.

Perpendicular Vectors

The vectors perpendicular to $\overrightarrow{AB} = \begin{pmatrix} a \\ b \end{pmatrix}$ are $\begin{pmatrix} b \\ -a \end{pmatrix}$ and $\begin{pmatrix} -b \\ a \end{pmatrix}$ or their scalar multiples $\begin{pmatrix} kb \\ -ka \end{pmatrix}$ and $\begin{pmatrix} -kb \\ ka \end{pmatrix}$.

For example: $\overrightarrow{PQ} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ is perpendicular to $\overrightarrow{RS} = \begin{pmatrix} -4 \\ -6 \end{pmatrix}$ when $\begin{pmatrix} -4 \\ -6 \end{pmatrix} = k\begin{pmatrix} -2 \\ -3 \end{pmatrix}$. That is $\begin{pmatrix} -4 \\ -6 \end{pmatrix} = \begin{pmatrix} -2k \\ -3k \end{pmatrix}$.

This implies that $-2k = -4 \Rightarrow k = 2$ or $-6 = -3k \Rightarrow k = 2$ (*i.e.*, the value of k must be the same).



This type of relationship may be represented in the above figure, where vector $\overrightarrow{AB}$ is perpendicular to vector $\overrightarrow{PQ}$.

Example 6. *Which of the vector is perpendicular to the vector* $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$?

(*a*) $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ (*b*) $\begin{pmatrix} -1 \\ -3 \end{pmatrix}$ (*c*) $\begin{pmatrix} 1 \\ -3 \end{pmatrix}$ (*d*) $\begin{pmatrix} -3 \\ -1 \end{pmatrix}$

Solution. The vectors perpendicular to $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$ are

$\begin{pmatrix} -3 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ or their scalar multiples

$\begin{pmatrix} -3k \\ -k \end{pmatrix}$ and $\begin{pmatrix} 3k \\ k \end{pmatrix}$ where k is a positive number. Therefore the correct option is (*d*).

EXERCISE 9.2

1. (*a*) If $\overrightarrow{AB} = \begin{pmatrix} -5 \\ 3 \end{pmatrix}$, find $\overrightarrow{BA}$ (*b*) If $\overrightarrow{AB} = \begin{pmatrix} 5 \\ 6 \end{pmatrix}$, find $\overrightarrow{BA}$.

2. (*a*) If $\overrightarrow{PQ} = \begin{pmatrix} 5 \\ 10 \end{pmatrix}$, find $\overrightarrow{QP}$ (*b*) If $\overrightarrow{DC} = \begin{pmatrix} -3 \\ 7 \end{pmatrix}$, find $-\overrightarrow{CD}$.

3. If P(–1, 3) and Q(2, 5), find $\overrightarrow{PQ}$.

4. If P(–6, 8) and Q(–10, 6), find $\overrightarrow{PQ}$.

5. Given that **s** = $\begin{pmatrix} x-4 \\ 3 \end{pmatrix}$ and **t** = $\begin{pmatrix} 5 \\ 3-y \end{pmatrix}$, find x and y if **s** = **t.**

6. If **a** = $\begin{pmatrix} 5-x \\ y+3 \end{pmatrix}$ and **b** = $\begin{pmatrix} -2 \\ -6 \end{pmatrix}$, find x and y when **a** = **b**.

7. Which of the following is parallel to $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$?

   (*a*) $\begin{pmatrix} 12 \\ 9 \end{pmatrix}$ (*b*) $\begin{pmatrix} -8 \\ 6 \end{pmatrix}$ (*c*) $\begin{pmatrix} 8 \\ 6 \end{pmatrix}$ (*d*) $\begin{pmatrix} -8 \\ -6 \end{pmatrix}$

8. If $\overrightarrow{PQ} = \begin{pmatrix} 24 \\ x \end{pmatrix}$ and $\overrightarrow{CD} = \begin{pmatrix} 16 \\ 20 \end{pmatrix}$, find x if $\overrightarrow{PQ}$ is parallel to $\overrightarrow{CD}$.

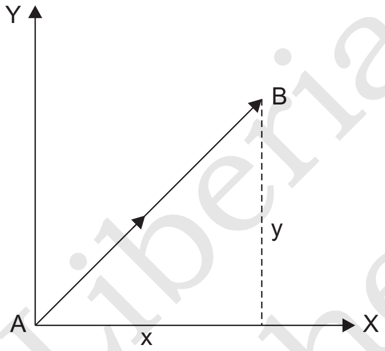
9. Which of the following vectors is perpendicular to the vector $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$?

   (*a*) $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$ (*b*) $\begin{pmatrix} 12 \\ 16 \end{pmatrix}$ (*c*) $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$ (*d*) $\begin{pmatrix} 8 \\ -6 \end{pmatrix}$

10. The vector $\begin{pmatrix} 9 \\ x \end{pmatrix}$ is perpendicular to the vector $\begin{pmatrix} 6 \\ 3 \end{pmatrix}$. Find the value of x.

9.3 MAGNITUDE *AND* DIRECTION OF A VECTOR

Magnitude or length of a vector



If $\vec{AB} = \begin{pmatrix} x \\ y \end{pmatrix}$, then the magnitude of $\vec{AB}$ denoted by $|\vec{AB}|$ or AB is given by:

$$|\vec{AB}| = AB = \sqrt{x^2 + y^2}$$

This is found by using Pythagoras theorem.

Exmaple 7. *If* $\vec{CD} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$ *and* $\vec{AB} = \begin{pmatrix} 5 \\ 12 \end{pmatrix}$, *find* $|\vec{CD}|$ *and* $|\vec{AB}|$.

Solution.

$$|\vec{CD}| = \sqrt{(-3)^2 + (4)^2}$$
$$= \sqrt{9+16} = \sqrt{25} = 5 \text{ units}$$

$$|\vec{AB}| = \sqrt{(5)^2 + (12)^2}$$
$$= \sqrt{(25+144)} = \sqrt{169} = 13 \text{ units.}$$

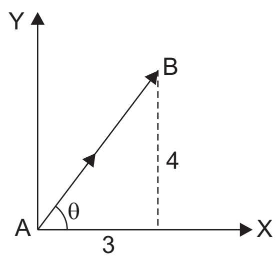
Example 8. *Find the length of the vector* $\vec{AB} = \begin{pmatrix} -5 \\ 10 \end{pmatrix}$.

Solution. Length of vector AB = $|\vec{AB}|$

$$\therefore \quad |\vec{AB}| = \sqrt{(-5)^2 + (10)^2}$$
$$= \sqrt{25+100} = \sqrt{125} = 5\sqrt{5} \text{ units.}$$

Direction of a Vector

The direction is measured from the *north* in the *clockwise direction.*



To find the direction of a vector, $\vec{AB} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$, first represent the vector on a rough diagram, and find the acute angle it makes with the x-axis using the relation for tangent of an angle in a right-angled triangle. Note that both the x and y components of the vector are positive meaning the vector is in the first quadrant.

From the diagram,

$$\tan\theta = \frac{4}{3} = 1.33 \quad \Rightarrow \quad \theta = 53°. \text{ [Using table of tangents of angles]}$$

$\therefore$ The direction of $\overrightarrow{AB}$ measured from the north

$= 90° - 53° = 37°.$

Example 9. *Find the magnitude and directions (bearings) of the following column vectors.*

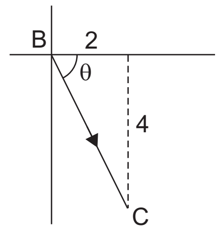
(*i*) $\overrightarrow{BC} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$ (*ii*) $\overrightarrow{PQ} = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$ (*iii*) $\overrightarrow{QR} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$

Note: Bearings give directions in terms of an angle.

Solution. (*i*) $|\overrightarrow{BC}| = \sqrt{(2)^2 + (-4)^2}$

$= \sqrt{4+16} = \sqrt{20} = 4.47$ units

For the direction of $\overrightarrow{BC}$, first represent the vector on a rough diagram, and find the acute angle it makes with the *x*-axis using the relation for tangent of an angle in a right angled triangle.



From the diagram, $\tan\theta = \frac{4}{2} = 2$

$\Rightarrow$ $\theta = 63.4°$ [Using table of tangents of angles]

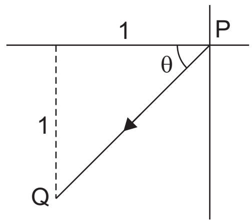
$\therefore$ The direction of $\overrightarrow{BC}$ measured from the north

$= 90° + 63.4° = 153.4°.$

(*ii*) $|\overrightarrow{PQ}| = \sqrt{(-1)^2 + (-1)^2}$

$= \sqrt{(1+1)} = \sqrt{2} = 1.41$ units

For the direction of $\overrightarrow{PQ}$, first represent the vector on a rough diagram, and find the acute angle it makes with the *x*-axis using the relation for tangent of an angle in a right-angled triangle.



From the diagram, $\tan\theta = \frac{1}{1} = 1$

$\Rightarrow \quad \theta = 45°$ [Using table of tangents of angles]

$\therefore$ The direction of $\overrightarrow{PQ}$ measured from the north

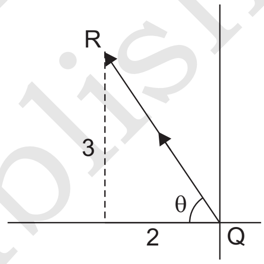
$= 270° - 45° = 225°$

(*iii*) $|\overrightarrow{QR}| = \sqrt{(-2)^2 + (3)^2}$

$= \sqrt{(4+9)} = \sqrt{13} = 3.6$ units.

For the direction of $\overrightarrow{QR}$, first represent the vector on a rough diagram, and find the acute angle it makes with the *x*-axis.

Using the relation for tangent of an angle in a right-angled triangle.



From the diagram, $\tan\theta = \frac{3}{2} = 1.5$

$\Rightarrow \quad \theta = 56°$ to the nearest degree. [Using table of tangents of angles]

$\therefore$ The direction of $\overrightarrow{QR}$ measured from the north

$= 270° + 56° = 326°$.

EXERCISE 9.3

1. Find the magnitude of the following vectors:

(*a*) $\overrightarrow{AB} = \begin{pmatrix} 12 \\ 13 \end{pmatrix}$ (*b*) $\overrightarrow{CD} = \begin{pmatrix} -3 \\ -4 \end{pmatrix}$ (*c*) $\overrightarrow{EF} = \begin{pmatrix} -5 \\ 12 \end{pmatrix}$ (*d*) $\overrightarrow{PQ} = \begin{pmatrix} 9 \\ 12 \end{pmatrix}$

2. Given the vector $\overrightarrow{PQ} = \begin{pmatrix} -7 \\ 12 \end{pmatrix}$, calculate the:

(*a*) length $|\overrightarrow{PQ}|$, correct to three significant figures

(*b*) bearing of Q from P, correct to the nearest degree.

3. If P(–1, 2) and Q(*x*, *y*) are point in the O*xy* plane such that $\overrightarrow{QP} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$. Find

(*a*) the co-ordinate of Q (*b*) $|\overrightarrow{QP}|$

(*c*) the bearing of Q from P.

4. If P(2, 2) and Q(5, 4)

(*a*) Calculate the magnitude of $\overrightarrow{PQ}$.

(*b*) Express $\overrightarrow{PQ}$ in the form (k, θ), where k is the magnitude and θ the bearing.

9.4 ADDITION *AND* SUBTRACTION OF VECTORS

ACTIVITY 1

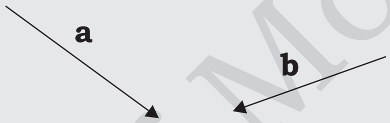
To add **a** and **b**:

Step 1: Draw **a.**

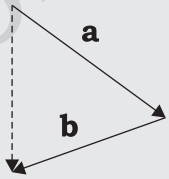
Step 2: At the arrowhead end of **a**, draw **b**.

Step 3: Join the beginning of **a** to the arrowhead end of **b**. This is vector **a + b**.

So, given



we have **a + b**



For example: If $\mathbf{a} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$, then

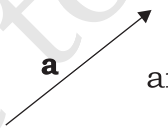
$$\mathbf{a} + \mathbf{b} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} + \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ y_1 + y_2 \end{pmatrix}$$

[Add the corresponding components]

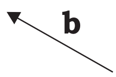
To subtract one vector from another, we simply *add its negative.*

$$\mathbf{a} - \mathbf{b} = \mathbf{a} + (-\mathbf{b})$$

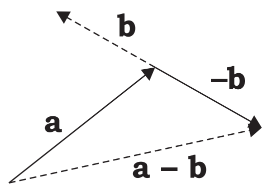
If we have



and



then



For example: If $\mathbf{a} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$, then

$$\mathbf{a} - \mathbf{b} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} - \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} x_1 - x_2 \\ y_1 - y_2 \end{pmatrix}$$

[Subtract the corresponding components]

Note: While adding two vectors, the end point of the first vector must be the same as the starting point of the second vector.

Example 10. *If* $\mathbf{r} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ *and* $\mathbf{s} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, *then find:*

(*i*) $\mathbf{r} + \mathbf{s}$ (*ii*) $\mathbf{r} - \mathbf{s}$

Solution. (*i*) $\mathbf{r} + \mathbf{s} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 3+1 \\ 4+2 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$

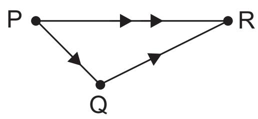
[Add the corresponding components]

(*ii*) $\mathbf{r} - \mathbf{s} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 3-1 \\ 4-2 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$

[Subtract the corresponding components]

Example 11. *If* $\vec{PQ} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$ *and* $\vec{RQ} = \begin{pmatrix} -5 \\ -8 \end{pmatrix}$, *find* $\vec{PR}$.

Solution. From the given vectors, $\vec{PR} = \vec{PQ} + \vec{QR}$

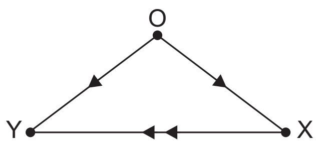


$\vec{PQ}$ has been given but $\vec{QR} = -\vec{RQ} = \begin{pmatrix} 5 \\ 8 \end{pmatrix}$

$\therefore \quad \vec{PR} = \begin{pmatrix} -1 \\ 3 \end{pmatrix} + \begin{pmatrix} 5 \\ 8 \end{pmatrix} = \begin{pmatrix} 4 \\ 11 \end{pmatrix}.$

Example 12. *If* $\vec{OX} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ *and* $\vec{OY} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$, *find* $|XY|$.

Solution. From the given vectors, $\vec{XY} = \vec{XO} + \vec{OY}$



$\vec{OY}$ has been given

But $\quad \vec{XO} = -\vec{OX} = \begin{pmatrix} -4 \\ -3 \end{pmatrix}$ [Negative vectors]

$\Rightarrow \quad \vec{XY} = \begin{pmatrix} -4 \\ -3 \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} -2 \\ -4 \end{pmatrix}$

$\therefore \quad |XY| = \sqrt{(-2)^2 + (-4)^2}$

$= \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$ units.

9.5 SCALAR MULTIPLICATION

If $\vec{AB} = \begin{pmatrix} x \\ y \end{pmatrix}$, then $k\,\vec{AB} = \begin{pmatrix} kx \\ ky \end{pmatrix}$, where k is a *scalar* or *number* which can be a negative or positive whole number or fraction. When k is positive, it implies the vectors are parallel and in the same direction. When k is negative, it implies the vectors are parallel but in opposite directions. The length of the new vector is $|\ k\ |$ times the length of the original vector.

Note: To find the scalar multiple of a vector, multiply each component of the vector by the scalar.

Example 13. *If* $\vec{AB} = \begin{pmatrix} -1 \\ 5 \end{pmatrix}$, *find (i)* $2\vec{AB}$ *(ii)* $-3\vec{AB}$.

Solution. (*i*) $2\vec{AB} = 2\begin{pmatrix} -1 \\ 5 \end{pmatrix} = \begin{pmatrix} 2\times(-1) \\ 2\times 5 \end{pmatrix} = \begin{pmatrix} -2 \\ 10 \end{pmatrix}$

(*ii*) $-3\vec{AB} = -3\begin{pmatrix} -1 \\ 5 \end{pmatrix} = \begin{pmatrix} -3\times(-1) \\ -3\times 5 \end{pmatrix} = \begin{pmatrix} 3 \\ -15 \end{pmatrix}$.

Example 14. *If* $\mathbf{a} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 4 \\ -5 \end{pmatrix}$ *and* $\mathbf{c} = \begin{pmatrix} -2 \\ -6 \end{pmatrix}$, *find*

(*i*) $2\mathbf{b}$ (*ii*) $\mathbf{a} - 2\mathbf{b} + \mathbf{c}$ (*iii*) $2(\mathbf{a} + \mathbf{b})$

Solution. (*i*) $2\mathbf{b} = 2\begin{pmatrix} 4 \\ -5 \end{pmatrix} = \begin{pmatrix} 8 \\ -10 \end{pmatrix}$

(*ii*) $\mathbf{a} - 2\mathbf{b} + \mathbf{c} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} - 2\begin{pmatrix} 4 \\ -5 \end{pmatrix} + \begin{pmatrix} -2 \\ -6 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} - \begin{pmatrix} 8 \\ -10 \end{pmatrix} + \begin{pmatrix} -2 \\ -6 \end{pmatrix}$

$$= \begin{pmatrix} 3 - 8 + (-2) \\ 1 - (-10) + (-6) \end{pmatrix} = \begin{pmatrix} -7 \\ 5 \end{pmatrix}.$$

(*iii*) First find the expression in the bracket i.e. $\mathbf{a} + \mathbf{b}$

$$\mathbf{a} + \mathbf{b} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} + \begin{pmatrix} 4 \\ -5 \end{pmatrix} = \begin{pmatrix} 7 \\ -4 \end{pmatrix}$$

$$\therefore \quad 2(\mathbf{a} + \mathbf{b}) = 2\begin{pmatrix} 7 \\ -4 \end{pmatrix} = \begin{pmatrix} 14 \\ -8 \end{pmatrix}.$$

EXERCISE 9.4

1. If $\mathbf{a} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$, find (*a*) $\mathbf{a} + \mathbf{b}$ (*b*) $\mathbf{a} - \mathbf{b}$.

2. Given that $\mathbf{a} = \begin{pmatrix} 2 \\ 7 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} -3 \\ -5 \end{pmatrix}$, find (*a*) $\mathbf{a} + \mathbf{b} - \mathbf{c}$ (*b*) $\mathbf{b} - \mathbf{c} + \mathbf{a}$.

3. If $\overrightarrow{AB} = \begin{pmatrix} -1 \\ -3 \end{pmatrix}$ and $\overrightarrow{BC} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$, find $\overrightarrow{AC}$.

4. If $\overrightarrow{XY} = \begin{pmatrix} -2 \\ -4 \end{pmatrix}$ and $\overrightarrow{ZY} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$, find $\overrightarrow{XZ}$.

5. Given that $\overrightarrow{PQ} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$ and $\overrightarrow{RQ} = \begin{pmatrix} 6 \\ -5 \end{pmatrix}$, find $|\overrightarrow{PR}|$.

6. If $\mathbf{r} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ and $\mathbf{s} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$, calculate $6(\mathbf{r} + 2\mathbf{s})$.

7. If $\mathbf{p} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $\mathbf{q} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, find $2\mathbf{p} - \mathbf{q} + \mathbf{r}$.

8. If $\mathbf{r} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$, $\mathbf{s} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$ and $\mathbf{t} = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$, find $2\mathbf{r} - \mathbf{s} + \mathbf{t}$.

REVIEW EXERCISE

1. Express the following vectors graphically.

   (*a*) $\overrightarrow{AB}$ = (3 km, 245°)   (*b*) $\overrightarrow{BC} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$

2. If $\boldsymbol{p} = \begin{pmatrix} 3x+1 \\ 2y+3 \end{pmatrix}$ and $\boldsymbol{q} = \begin{pmatrix} -8 \\ 5 \end{pmatrix}$, find x and y, if $\boldsymbol{p} = \boldsymbol{q}$.

3. If $\overrightarrow{BA} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$, find $\overrightarrow{AB}$.

4. If $\mathbf{p} = \begin{pmatrix} 4x+3 \\ 5+2y \end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix} 11 \\ -5 \end{pmatrix}$, find x and y when $\mathbf{p} = \mathbf{q}$.

5. Which of the following is parallel and opposite to $\begin{pmatrix} 2 \\ -6 \end{pmatrix}$?

   (*a*) $\begin{pmatrix} 4 \\ 12 \end{pmatrix}$ (*b*) $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$ (*c*) $\begin{pmatrix} 1 \\ -3 \end{pmatrix}$ (*d*) $\begin{pmatrix} -4 \\ -12 \end{pmatrix}$

6. The vector $\begin{pmatrix} 12 \\ x \end{pmatrix}$ is parallel to the vector $\begin{pmatrix} 8 \\ 6 \end{pmatrix}$. Find the value of x.

7. Find the magnitude of the following vectors:

(*a*) $\vec{LM} = \begin{pmatrix} -7 \\ 24 \end{pmatrix}$ (*b*) $\vec{XY} = \begin{pmatrix} 15 \\ 8 \end{pmatrix}$

8. If $\mathbf{a} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 4 \\ -5 \end{pmatrix}$ then, find (*a*) $\mathbf{a} + \mathbf{b}$ (*b*) $\mathbf{a} - \mathbf{b}$.

9. If $\vec{XY} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$ and $\vec{ZY} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$, find $\vec{XZ}$.

10. Given that $\mathbf{p} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix} 8 \\ 6 \end{pmatrix}$, find (*a*) $\mathbf{p} + \mathbf{q}$ (*b*) $\mathbf{q} - \mathbf{p}$.

11. If $\vec{PQ} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$ and $\vec{RQ} = \begin{pmatrix} -1 \\ -4 \end{pmatrix}$, find $\vec{PR}$.

12. Given that $\vec{PQ} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ and $\vec{RQ} = \begin{pmatrix} -3 \\ -5 \end{pmatrix}$, find $\vec{PR}$.

13. If $\mathbf{a} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$, then find (*a*) $-3\mathbf{a}$ (*b*) $2\mathbf{a}$

14. If $\mathbf{a} = \begin{pmatrix} 3 \\ -6 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, find $2\mathbf{a} + 3\mathbf{b}$.

MULTIPLE CHOICE QUESTIONS (MCQs)

1. Simplify $\begin{pmatrix} 2 \\ -3 \end{pmatrix} - \begin{pmatrix} -1 \\ 5 \end{pmatrix}$

(a) $\begin{pmatrix} -3 \\ 8 \end{pmatrix}$ (b) $\begin{pmatrix} 3 \\ -8 \end{pmatrix}$ (c) $\begin{pmatrix} -3 \\ -8 \end{pmatrix}$ (d) $\begin{pmatrix} 3 \\ 8 \end{pmatrix}$

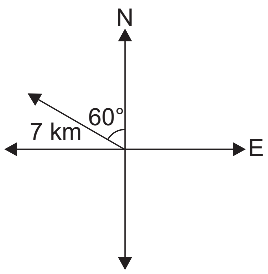
2. If $\mathbf{r} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ and $\mathbf{s} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$, calculate $6(\mathbf{r} + 2\mathbf{s})$

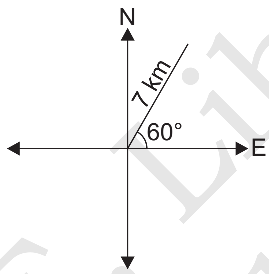
(a) $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$ (b) $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ (c) $\begin{pmatrix} 7 \\ 3 \end{pmatrix}$ (d) $\begin{pmatrix} -6 \\ 18 \end{pmatrix}$

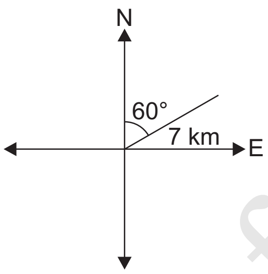
3. Find the length of the vector P = $\begin{pmatrix} -5 \\ 12 \end{pmatrix}$

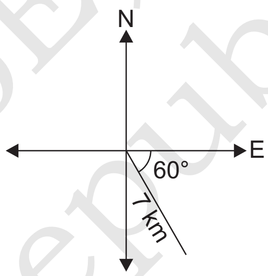
(a) 17 (b) 13 (c) 25 (d) 7

4. A boy walked 7 km on a bearing 60°. Which of the following diagrams shows his direction?

(a) 

(b) 

(c) 

(d) 

5. If P(2, 5) and Q(–2, 3) are points in the cartesian plain, find the vector $\overrightarrow{PQ}$.

(a) $\begin{pmatrix} 4 \\ -2 \end{pmatrix}$ (b) $\begin{pmatrix} -4 \\ -2 \end{pmatrix}$ (c) $\begin{pmatrix} -4 \\ 2 \end{pmatrix}$ (d) $\begin{pmatrix} 4 \\ 2 \end{pmatrix}$

6. Find k in the vector equation, $\begin{pmatrix} 3 \\ 4 \end{pmatrix} + k\begin{pmatrix} 3 \\ 4 \end{pmatrix} = -\begin{pmatrix} 3 \\ 4 \end{pmatrix}$

(a) $-\frac{3}{4}$ (b) –3 (c) 2 (d) –2

7. Simplify $\begin{pmatrix} 4 \\ 5 \end{pmatrix} + \begin{pmatrix} -2 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ 6 \end{pmatrix}$

(a) $\begin{pmatrix} 7 \\ 13 \end{pmatrix}$ (b) $\begin{pmatrix} 5 \\ 13 \end{pmatrix}$ (c) $\begin{pmatrix} 2 \\ 13 \end{pmatrix}$ (d) $\begin{pmatrix} 3 \\ 13 \end{pmatrix}$

8. If $\mathbf{r} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\mathbf{s} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$. Calculate $2\mathbf{r} - 3\mathbf{s}$.

(a) $\begin{pmatrix} 18 \\ -8 \end{pmatrix}$ (b) $\begin{pmatrix} -5 \\ 12 \end{pmatrix}$ (c) $\begin{pmatrix} 13 \\ -4 \end{pmatrix}$ (d) $\begin{pmatrix} -5 \\ -8 \end{pmatrix}$

9. If $\mathbf{u} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$, find $2\mathbf{u} + 3\mathbf{v}$.

(a) $\begin{pmatrix} 11 \\ 5 \end{pmatrix}$ (b) $\begin{pmatrix} 11 \\ 13 \end{pmatrix}$ (c) $\begin{pmatrix} 5 \\ 1 \end{pmatrix}$ (d) $\begin{pmatrix} 14 \\ 0 \end{pmatrix}$

10. If $\mathbf{u} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 6 \\ 1 \end{pmatrix}$, find $2\mathbf{u} + \mathbf{v}$.

(a) $\begin{pmatrix} 12 \\ 3 \end{pmatrix}$ (b) $\begin{pmatrix} 12 \\ -1 \end{pmatrix}$ (c) $\begin{pmatrix} 12 \\ 0 \end{pmatrix}$ (d) $\begin{pmatrix} 12 \\ 1 \end{pmatrix}$

RECAP AT A GLANCE

- A quantity that has only magnitude is called a *scalar.*
- A quantity that has magnitude as well as a direction is called a *vector.*
- A *vector* is any physical quantity that has *length* (or magnitude) and *direction.*
- A *zero vector* is a vector which *begins* and *ends* at the *same point.*
- A *position vector* is a vector which begins from the *origin* and ends at a point.
- Negative vector is the *opposite* or *inverse* of a vector.
- If any two or more vectors have the same components, then they are *equal vectors.*
- Two or more vectors are parallel, if one is a *scalar multiple* of the other(s).
- The vectors perpendicular to $\vec{AB} = \begin{pmatrix} a \\ b \end{pmatrix}$ are $\begin{pmatrix} b \\ -a \end{pmatrix}$ and $\begin{pmatrix} -b \\ a \end{pmatrix}$ or their scalar multiples $\begin{pmatrix} kb \\ -ka \end{pmatrix}$ and $\begin{pmatrix} -kb \\ ka \end{pmatrix}$.
- If $\vec{AB} = \begin{pmatrix} x \\ y \end{pmatrix}$, then the magnitude of $\vec{AB}$ denoted by $|\vec{AB}|$ or AB is given by:

$$|\vec{AB}| = AB = \sqrt{x^2 + y^2}$$

- To find the scalar multiple of a vector, multiply each component of the vector by the scalar.

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